Independent Samples: Comparing Proportions

Lecture 40
Section 11.5

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Fri, Apr 4, 2008

Outline

Introduction

- We continue to study the difference between population proportions, with an emphasis now on confidence intervals.

Turmoil at City Hall

Article

City Hall turmoil: Richmond Times-Dispatch poll

- In the survey, we had 240 males (48%) and 260 females (52%).
- 41% of the males, or 98 males, said Wilder is doing good or excellent.
- 37% of the females, or 96 females, said he is doing good or excellent.

Turmoil at City Hall

- The pooled estimate for p is

$$\hat{p} = \frac{98 + 96}{240 + 260} = \frac{194}{500} = 0.388.$$

- Now compute

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{(0.388)(0.612)\left(\frac{1}{240} + \frac{1}{260}\right)} = 0.04362.$$

- For comparison, estimate $\sigma_{\hat{p}_1 - \hat{p}_2}$ without using the pooled estimate. There will not be much difference. Nevertheless, we should always use the pooled estimate.

The Value of the Test Statistic

- Now compute z:

$$z = \frac{0.04}{0.04362} = 0.9170.$$

- Compute the p-value:

$$P(Z > 0.9170) = 0.1796.$$

- Accept H_0.
- Equal proportions of men and women believe that Mayor Wilder is doing a good or excellent job.

Turmoil at City Hall, Continued

- Do equal proportions of whites and blacks believe that Mayor Wilder is doing a good or excellent job?
- Do equal proportions of Republicans and Democrats believe that Mayor Wilder is doing a good or excellent job?
- Work the first one by hand and then let's learn how to do this on the TI-83 to work the other one.

TI-83 - Testing Hypotheses Concerning $p_1 - p_2$

- To test a hypothesis concerning $p_1 - p_2$ on the TI-83, do the following.
  - Press `STAT > TESTS > 2-PropZTest...`
  - Enter x_1
  - Enter n_1
  - Enter x_2
  - Enter n_2
  - Choose the correct alternative hypothesis.
  - Select `Calculate` and press `ENTER`.

TI-83 - Testing Hypotheses Concerning $p_1 - p_2$

- A window appears with the following information.
  - The title.
  - The alternative hypothesis.
  - The value of the test statistic z.
  - The p-value.
  - $\hat{p}_1$.
  - $\hat{p}_2$.
  - The pooled estimate $\hat{p}$.
  - n_1.
  - n_2.

Example

- Work “Turmoil at City Hall" again, using the TI-83.
- Test the hypothesis concerning Whites vs. Blacks, using the TI-83.
- Test the hypothesis concerning Republicans vs. Democrats, using the TI-83.

Confidence Intervals for $p_1 - p_2$

- The formula for a confidence interval for $p_1 - p_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

- Note that we do not use the pooled estimate for $\hat{p}$ because we are not assuming that $p_1 = p_2$.

TI-83 - Confidence Intervals for $p_1 - p_2$

- To find a confidence interval for $\hat{p}_1 - \hat{p}_2$ on the TI-83, do the following.
  - Press `STAT > TESTS > 2-PropZInt...`
  - Enter x_1
  - Enter n_1
  - Enter x_2
  - Enter n_2
  - The confidence level.
  - Select `Calculate` and press `ENTER`.

TI-83 - Confidence Intervals for $p_1 - p_2$

- A window appears with the following information.
  - The title.
  - The confidence interval.
  - $\hat{p}_1$.
  - $\hat{p}_2$.
  - n_1.
  - n_2.

Example

- Find a 95% confidence interval for the difference between the proportions of whites and blacks who believe that Mayor Wilder is doing a good or excellent job.

Summary

Summary

- When computing a confidence interval for $p_1 - p_2$, we do *not* pool the data.
- The formula for the confidence interval is

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$